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4 PhD Program & Course Schema 85

4.1 Graduate Program

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4.3 Distribution of Total Credit Hours

4. PhD Program & Course Schema

4.1 Graduate Program

4.1.1 PhD – Mathematics

In many disciplines, mathematics plays an extremely important role: ranging from stochastic analysis and models of large scale systems and networks, financial mathematics, statistical models, fluid dynamics computations, and risk assessment. Mathematical research, in general, contributes indirectly by supporting the advanced research in a variety of other disciplines offered at Sukkur IBA University.

The Department of Mathematics offers several courses for the students, such as Calculus, Linear Algebra, Analysis, Mathematical Modeling, Probability Theory, Mathematical finance and Statistics, Discrete Mathematics, and Stochastic processes. The offered coursework is designed to provide basic as well as advanced training for the tools of applied mathematics, including numerical methods, ordinary, partial and stochastic differential equations and scientific computing. The courses offered build a solid mathematical background of the students, to further pursue with their doctoral research/degree. The graduate program opens opportunities for the students interested in multidisciplinary mathematics projects for their PhD research in the fields of Mathematics.

Sukkur IBA University has about 17 faculty members working in different area of Mathematics. Mathematics department offers PhD courses, which are conducted by our HEC recognized PhD faculty. The PhD course contents are also designed such that students can improve and enhance their fundamental knowledge which is required to conduct doctoral research in their area of interest. The offered PhD coursework is designed to easily choose from the following research areas of the available faculty members:

- Mathematical Finance
- Abstract Mathematics
- Applied Mathematics
- Computational Mathematics
- Analysis

4.1.2 Scope regarding market, social and employment perspective of program

The logical thinking/reasoning and strategic knowledge of mathematics is highly valued by operational researchers, stockbrokers, business and investment analysts. Therefore, mathematicians are hired by a wide range of sectors in the market, for example by insurance companies, marketing companies, banks, and accountancy firms. Mathematicians have opportunities for employment in many engineering firms, industries, publicly funded research institutes or government agencies, and different government departments in Pakistan. Moreover, if one wants to pursue their career in academia then they can easily find a job in some reputed institutes or universities in Pakistan.

4.1.3 Program Goals & Objectives

The goals & objectives of our PhD program in Mathematics are as follows:

Goal 1 – Ethical Consciousness Becoming active in conducting the quality research with professional and ethical integrity.

Goal 2 – Scientific Consciousness
- Contributing in the development of new emerging fields in mathematics as well as managing the complex interdisciplinary projects.
- Develop highly trained mathematicians who can explore and address the mathematical needs in other sciences, industry and community.

Goal 3 – Critical Thinking Skills
- extending the mathematical thinking and proficiency beyond master level.
- develop a level of quantitative literacy and skills to effectively deal with the quantitative issues that they will encounter throughout their lives.

Goal 4 – Research Skills
- Provide training for the development of the research capabilities in mathematics.
- Building a strong foundation in mathematics analysis and its real world applications.

4.1.4 Eligibility Criteria for Admission

The students eligible for admittance in Ph.D program should have the following:

- MS/M.Phil degree with a minimum CGPA 3.0 (out of 4.0 in semester system) or first division (in the annual system).
- A subject test conducted by the National Testing Service (NTS) or ETS, USA in the area of specialization chosen at the PhD level must be cleared prior to admission for the PhD Program.
  - In case of GAT subject or GRE subject test, the minimum required score is 60% marks.
  - In case of STS, conducted by Sukkur IBA University, the minimum required score is 60% marks.

4.1.5 Degree Requirements

Duration of program: The minimum duration of completion of MS Program is 2 years (Four Semesters) and maximum duration is 4 years. Students are required to successfully qualify for eight courses each of 3 credit hours duration. On successful completion of MS course work students will be allowed to work on an 18– credit hour thesis on a subject of his/her interest.

Category	**Courses**	**Credit Hours**
Elective Courses	06	18
Thesis	– –	30
Total Credit Hours	– –	48

R Note that PhD scholars must fulfill the HEC minimum criteria for PhD degree. Details of HEC requirements are available at:
http://hec.gov.pk/english/scholarshipsgrants/Documents/MPHIL_Phd_Criteria.pdf.

4.2 Comprehensive Exam Policy

Comprehensive exam of PhD student will be conducted in two phases i.e. written and oral. The written exam will consist of two papers, namely, *i*) Core and *ii*) Specialization Courses. The duration of each written exam paper will be 03 hours. The weighting of these exams is as under:

# of Papers	Courses	Marks
Paper 1	Core	100
Paper 2	Specialization	100
Total	−−	200

1. Core exam (Paper 1) includes course on Calculus, Real Analysis, Complex Analysis, Linear Algebra, Group Theory, Ordinary and Partial Differential Equations, Numerical Analysis and Differential Geometry.
2. Specialization Courses (Paper 2) will be decided from a candidate's PhD course work as per the recommendations of the committee and/or a candidate's supervisor.
3. Each exam will be prepared as per the course outlines provided to candidates, in advance.
4. The course outlines for each course will be prepared by a designated senior PhD faculty member having expertise in that particular area.
5. A PhD comprehensive exam committee, composed of senior PhD faculty members, will be formed.
6. Each exam will be prepared and assessed by a designated senior PhD faculty member who will be a member of the comprehensive exam committee.
7. Each paper will be reviewed by the comprehensive exam committee and the oral exam will also be evaluated by the committee.
8. For passing the comprehensive exam, a candidate has to secure 60% marks in the each exam of Core courses (Paper−I) and Specializations courses (Paper−II).
9. Only two chances will be given to the candidate to qualify the comprehensive exam.

4.3 Distribution of Total Credit Hours

1st Year			Semester–I
S. No.	**Course Code**	**Course Title**	**Credit Hours**
1.	– – – – –	Elective – I	03
2.	– – – – –	Elective – II	03
3.	– – – – –	Elective – III	03
Total Credit Hours			09

1st Year			Semester–II
S. No.	**Course Code**	**Course Title**	**Credit Hours**
1.	– – – – –	Elective – IV	03
2.	– – – – –	Elective – V	03
3.	– – – – –	Elective – VI	03
Total Credit Hours			09

2nd Year			Semester–III
S. No.	**Course Code**	**Course Title**	**Credit Hours**
1.	– – – – –	Comprehensive Exam	–
2.	– – – – –	Research Proposal Defense	–
Total Credit Hours			

2nd Year			Semester–IV
S. No.	**Course Code**	**Course Title**	**Credit Hours**
1.	MAT–780	Research Work	
Total Credit Hours			

3rd Year			Semester–V
S. No.	**Course Code**	**Course Title**	**Credit Hours**
1.	MAT–780	Research Work	
Total Credit Hours			

3rd Year			Semester–VI
S. No.	**Course Code**	**Course Title**	**Credit Hours**
1.	MAT–780	Research Work	
Total Credit Hours			

4th Year			Semester–VII
S. No.	**Course Code**	**Course Title**	**Credit Hours**
1.	MAT–780	Research Work	
Total Credit Hours			

4th Year			Semester–VIII
S. No.	**Course Code**	**Course Title**	**Credit Hours**
1.	MAT–780	Thesis	30
Total Credit Hours			48

4.3.1 Elective Courses

> Note that the elective Courses will be offered on the availability of course instructors and the number of students registered for the course.

Mathematical Finance			
S. No.	**Course Code**	**Course Title**	**Credit Hours**
1.	MAF−751	Discrete Time Pricing of Derivative and Securities	03
2.	MAF−752	Modeling of Bonds, Term Structure, and Interest Rate Derivatives	03
3.	MAF−753	Stochastic Calculus & Black−Scholes Theory	03
4.	MAF−754	Bond Market and Interest Rate Modeling	03
5.	MAF−755	Credit Risk Modeling	03
6.	MAF−756	Mathematical Methods for Finance	03
7.	MAF−757	Stochastic Differential Equations	03
8.	MAF−758	Stochastic Partial Differential Equations	03
9.	MAF−759	Measure and Integration	03
10.	MAF−760	Stochastic Processes	03
11.	MAF−761	Actuarial Mathematics	03
12.	MAF−762	Portfolio Theory and Risk Management	03

List of Elective Courses in Mathematical Finance

Description of the Elective Courses in Mathematical Finance

MAF−751: Discrete Time Pricing of Derivative and Securities The aim of the course is to explain in simple - namely discrete time - settings the fundamental ideas of modeling of financial markets and pricing of derivative securities using the principle of no arbitrage. Even the simplest of all models with only a one-time step allows several important notions to be illustrated. The course progresses with more complex models – with many time steps and several stocks – which are developed along with the corresponding theory of pricing and hedging derivative securities such as options or forwards. Relatively simple mathematical considerations lead to powerful notions and techniques underlying the theory – such as viability, completeness, self-financing and replicating strategies, arbitrage and equivalent martingale measures. These are directly applicable in practice, particularly in the continuous time limiting theory developed in a subsequent course. The general methods are applied in detail in particular to pricing and hedging European and American options within the Cox-Ross-Rubinstein (CRR) binomial tree model. The Black-Scholes model as the limit of CRR models is discussed to pave the way for continuous time theory.

Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. M. Capinski and T. Zastawniak. *Mathematics for Finance: An Introduction to Financial Engineering*. Springer, 2003
2. Marek Capinski and Ekkehard Kopp. *Discrete Models of Financial Markets*. Cambridge University Press, 2012
3. N. Cutland and A. Roux. *Derivative Pricing in Discrete Time*. 2nd edition. Springer-Verlag, 2012

MAF−752: Modeling of Bonds, Term Structure, and Inter-est Rate Derivatives The course introduces the probabilistic concepts and techniques necessary for modeling the dynamics of interest rates. The mathematical theory of interest rates is complex because it has to cover random behavior of several different rates simultaneously, while remaining consistent with no-arbitrage restrictions. Additionally, to be realistic, models need to allow calibration of the parameters to real data. The complexity stems from the fact that in general interest rates

depend on running time and maturity time, so are stochastic processes of two-time variables, each with a very specific role. Discrete models will be constructed based on tree structures. In full generality the theory of partial stochastic differential equations is needed to investigate continuous-time models.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. M. Capinski and T. Zastawniak. *Mathematics for Finance: An Introduction to Financial Engineering*. Springer, 2003
2. D. McInerney and T. Zastawniak. *Stochastic Interest Rates*. Cambridge University Press, 2015
3. T. Bjork. *Arbitrage Theory in Continuous Time*. Oxford University Press, 2004

MAF−753: Stochastic Calculus & Black-Scholes Theory This course enables students to acquire in-depth knowledge of the main features of Ito stochastic calculus as applied in mathematical finance, including: The role of the Ito integral and Ito formula in solving stochastic differential equations (SDEs). Martingale properties of the Ito integral and the structure of Brownian martingales. The mathematical relationships between wealth processes, investment strategies and option prices. Change of measure techniques and Girsanov's theorem. Partial differential equation (PDE) approach, and in particular the Black-Scholes equation. Feynman-Kac representation of option prices. The emphasis is on fundamental concepts which underlie the main continuous-time models of option pricing, principally the Black-Scholes model. Both plain vanilla (European) and exotic options (for example, barrier options) are dealt with, and the relationship between the approaches based on martingale theory and partial differential equations is explored. The course aims to equip students with a thorough understanding of the sophisticated mathematical results and techniques encountered in financial market modeling.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. M. Capinski, E. Kopp, and J. Traple. *Stochastic Calculus for Finance*. Cambridge University Press, 2012
2. M. Capinski and E. Kopp. *The Black-Scholes Model*. Cambridge University Press, 2012
3. R. A. Dana and M. Jeanblanc. *Financial Markets in Continuous Time*. Springer, 2001

MAF−754: Bond Market and Interest Rate Modeling The course introduces the probabilistic concepts and techniques necessary for modeling the dynamics of interest rates. The mathematical theory of interest rates is complex because it has to cover random behavior of several different rates simultaneously, while remaining consistent with no-arbitrage restrictions. Additionally, to be realistic, models need to allow calibration of the parameters to real data. The complexity stems from the fact that in general interest rates depend on running time and maturity time, so are stochastic processes of two time variables, each with a very specific role. Discrete models will be constructed based on tree structures. In full generality the theory of partial stochastic differential equations is needed to investigate continuous-time models.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. M. Capinski and T. Zastawniak. *Mathematics for Finance: An Introduction to Financial Engineering*. Springer, 2003
2. D. McInerney and T. Zastawniak. *Stochastic Interest Rates*. Cambridge University Press, 2015

MAF−755: Credit Risk Modeling The course aims to acquaint students with modern mathematical theory of credit risk and make them aware of its important applications in post-credit crisis financial markets. The course will focus on the two mainstream modeling approaches to credit risk, namely structural models and reduced form models, and pricing selected credit risk derivatives. Key contents include: Merton's structural model, Barrier model, Hazard function model and no arbitrage, Defaultable bond pricing with hazard function, Pricing of securities with hazard function, Hazard process model, Pricing of defaultable securities within the hazard process model

Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. M. Capinski and T. Zastawniak. *Credit Risk, Mastering Mathematical Finance Series*. Cambridge University Press, 2015

MAF−756: Mathematical Methods for Finance Modern finance speaks the language of the stochastic calculus of random processes due to its inherent randomness of asset prices. Since the 'pre-calculus' for stochastic calculus is probability theory, the aim of this course is to provide all theoretical foundations of probability to begin with stochastic calculus and hence modern quantitative finance. The course will begin with the review of the elementary review of the probability theory with focus conditions expectations. Then the course will proceed towards exploring the Wiener, Poisson and random walks through several numerical examples, graphical illustrations and theorems. By the end of this course students are expected to achieve a sufficient level of competence in selected mathematical methods and techniques to facilitate further study of Mathematical Finance.

Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. E. Kopp, J. Malczak, and T. Zastawniak. *Probability for Finance*. Cambridge University Press, 2013
2. M. Capinski and T. Zastawniak. *Mathematics for Finance: An Introduction to Financial Engineering*. Springer, 2003

MAF−757: Stochastic Differential Equations This course gives an introduction to the theory of stochastic differential equations (SDEs), explains real-life applications, and introduces numerical methods to solve these equations. The key Topics include Brownian motion and Langevin's equation. Ito and Stratonovich Stochastic integrals. Stochastic calculus and Ito's formula. SDEs and PDEs of Kolmogorov. Fokker-Planck, and Dynkin. Boundary conditions, exit times, exit distributions, stability. Asymptotic analysis of SDE, the Smoluchowski-Kramers approximation, diffusion approximation to Markov chains. Applications.

Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. **Evans2012**

MAF−758: Stochastic Partial Differential Equations This course introduces various methods for understanding solutions and dynamical behaviors of stochastic partial differential equations arising from mathematical modeling in science and engineering and other areas. It is designed for graduate students who would like to use stochastic methods in their research or to learn such methods for long term career development. Topics include: Random variables, Brownian motion and stochastic calculus in Hilbert spaces; Stochastic heat equation; Stochastic wave equation; Analytical and approximation techniques; Stochastic numerical simulations via Matlab; Dynamical impact of noises; Stochastic flows and cocycles; Invariant measures, Lyapunov exponents and ergodicity; and applications to engineering and science and other areas.

Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. Wei Liu and Michael Röckner. *Stochastic Partial Differential Equations: An Introduction*. Springer, 2015

MAF−759: Measure and Integration The aim of the course is to give a solid presentation of Measure and Integration. The theory of measure and integration gives an academic basis for studying Probability and Functional Analysis, and partial differential equations. The following main topics are contained in the course: Sigma-algebras and measures, measurable mappings, integration with respect to measures, the Lebesgue measure on the real line and on Rk, product measures, Lp-spaces.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. R. Schilling. *measure probability and martingales*. 1st edition. Cambridge University Press, 2005

MAF−760: Stochastic Processes This course prepares students to a rigorous study of Stochastic Calculus and differential equations. Main topics are introduction to measurable spaces, measures random variables and (conditional) expectation, uniform integrability and modes of convergence, stationary and sample path continuity of stochastic processes, examples such as Markov chains, Branching, Gaussian and Poisson Processes, Martingales and basic properties of Brownian motion.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. Zdzisław Brzezniak and Tomasz Zastawniak. *Basic Stochastic Processes. A Course Through Exercises*. Springer Science & Business Media, 1999

MAF−761: Actuarial Mathematics Probability and Statistics The course of the Actuarial mathematics is all about to provide students with a solid background in the subject of life contingencies for a single (and if time permitted multiple) life, and experience of its application to the analysis of life assurance and life annuity (including pension) contracts. The course includes the topics: Survival models; Life Tables; Life insurance; Life annuities; Benefit premiums Benefit reserves; Pension mathematics.
Pre-requisite: Probability and Statistics is the pre-requisites for this course.
Recommended Books

1. David C. M. Dickson and Mary R. Hardy. *Actuarial Mathematics for Life Contingent Risks*. Camb. Uni. Press., 2010

MAF−762: Portfolio Theory and Risk Management Students are expected to acquire the skills and knowledge necessary to apply modern risk measures and management tools and to use portfolio theory to manage and balance investment risk and return. The main emphasis here is on employing the concept of diversification for management of stock investment. A more general approach involves utility functions and the construction of portfolios using expected utility optimization. Portfolios of various derivative securities are powerful tools for risk management. Sophisticated needs of fund managers can be addressed by designing these portfolios. Students also need to demonstrate familiarity with pricing models along with their strengths and disadvantages. The topic includes: Value at Risk (VaR); Mean and variance as a measure of return and risk; Risk and return of a portfolio of two assets, diversification. Construction of the feasible set; Risk minimization for two assets. Indifference curves, optimization of portfolio selection based on individual preferences; Inclusion of risk free asset. Finding the market portfolio in two-assets market; Discussion of the separation principle (single fund theorem); Market imperfections; different rates for borrowing and lending; Non-linear optimization: Lagrange multipliers; General case of many

assets, risk-minimization, efficient frontier and its characterization; Market imperfections: no short-selling; Capital Asset Pricing Model, Security Market Line; practical applications – equilibrium theory; Certainty equivalent form of CAPM; Arbitrage Pricing Theory.
Pre-requisite: Probability and Statistics is the pre-requisites for this course.

Recommended Books

1. D. G. Luenberger. *Investment Science*. 1st edition. Oxford University Press, 1998
2. D. Duffie. *Dynamic Asset Pricing Theory*. 1st edition. Princeton University, 2001
3. T.E. Copeland and J.F. Weston. *Financial Theory and Corporate Policy*. 1st edition. Addison Wesley, 1992

List of Elective Courses in Abstract Mathematics

Abstract Mathematics			
S. No.	**Course Code**	**Course Title**	**Credit Hours**
1.	ABM−751	Theory of Group Actions and Group Graphs	03
2.	ABM−752	Homological Algebra	03
3.	ABM−753	Error Correction and Coding theory	03
4.	ABM−754	Theory of Semigroups	03
5.	ABM−755	Universal Algebra	03
6.	ABM−756	Category Theory	03
7.	ABM−757	Inverse Semigroups: The Theory of Partial Symmetries	03
8.	ABM−758	Advanced Topics in Group Theory	03
9.	ABM−759	Representation Theory of Algebraic Groups	03
10.	ABM−760	Algebraic Geometry	03
11.	ABM−761	Linear Algebraic Groups	03
12.	ABM−762	Applied Group Theory	03
13.	ABM−763	Commutative Algebra	03
14.	ABM−764	Computational Algebra and Geometry	03
15.	ABM−765	Coxeter Groups	03
16.	ABM−766	Applied Algebra	03
17.	ABM−767	Representation Theory of Finite Groups	03
18.	ABM−768	Lie Algebras	03
19.	ABM−769	Formal Languages and Automata	03

Description of the Elective Courses in Abstract Mathematics

ABM−751: Theory of Group Actions and Group Graphs The aim of this course is to introduce the fundamental methods and problems of geometric group theory and discuss their relationship to topology and geometry. It begins with action of a group on a set and continues with the theory of group actions on trees and the structural study of fundamental groups of graphs of groups. Topics include: Introduction and examples, Orbits and stabilizers, Actions of P-groups, new proofs using P-groups, Symmetric and alternating group, Applications of group actions to group theory, Semidirect products, Introduction to graphs, Cayley graphs: introduction, recognition, examples, isomorphisms, automorphisms, Subgraphs, factorisations, embeddings, applications, Computing with graphs and groups, Coset enumeration, Coset enumeration for symmetric graphs.

Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. John Meier. *Graphs, groups and trees*. Cambridge University Press, 2008
2. Israel Grossman and Wilhelm Magnus. *Groups and Their Graphs*. Mathematical Association of America, 1992

ABM−752: Homological Algebra Homological algebra is a branch of mathematics that studies homology in a general algebraic setting. Homological algebra established itself as a separate branch of mathematics around the time of WWII. Nowadays it is a profound branch of mathematics and an essential tool. For example, the study of class field theory relies crucially on homological algebra. Main topics of this course include Modules; categories and functors; extension of modules; derived functors; The Kunneth Formula; cohomology of groups; cohomology of Lie Algberas; exact couples and spectral sequences; Satellites and homology;

some applications and recent developments.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. Peter J. Hilton and Urs Stammbach. *A course in Homological Algebra*. Springer Science & Business Media, 1971

ABM−753: Error Correction and Coding theory Coding theory originated in the late 1940's and took its roots in engineering. However, it has developed and become a part of mathematics, and especially computer science. Codes were initially developed to correct errors on noisy and inaccurate communication channels. In this endeavor, linear codes are very helpful. Linear codes are simple to understand, yet are the most important and affective for practical applications, such as encoding and decoding messages sent over communication channels. More specially, we s examine Hamming codes and their properties. Furthermore, we shall discuss the importance of finite field to generate a cyclic and BCH codes.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. Peter J. Hilton and Urs Stammbach. *A course in Homological Algebra*. Springer Science & Business Media, 1971
2. W. Cary Huffman and Vera Pless. *Fundamentals of Error-Correcting Codes*. Cambridge University Press, 2003

ABM−754: Theory of Semigroups The algebraic theory of semigroups is relatively new in modern pure mathematics. It began slowly in the early decades of the last century to develop results for algebraic theories of groups and rings. But from 1950s this theory blossomed into new directions. Semigroups appear and are used in many mathematical areas other than algebra, such as, theoretical computer science, category theory, and analysis and number theory. In mathematical terms, a semigroup is an algebraic structure consisting of a set together with an associative binary operation. Topics include: Basic concepts, examples, Standard algebraic tools, morphisms and transformations, partial orders and semilattices, Idempotents, semilattices and bands, Quotients, homomorphisms, ideals and Rees congruences, Free semigroups and presentations, Green's relations, Green's Lemma and Green's Theorem. Structure of regular D-classes, Rees matrix semigroups, 0-simple and completely 0-simple semigroups, Regular semigroups, inverse semigroups, representations by injective partial mappings, congruences of inverse semigroups, free inverse semigroups, Semigroups decomposing tools.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. John Mackintosh Howie. *Fundamentals of Semigroup Theory*. 2nd edition. Clarendon Press, 1995

ABM−755: Universal Algebra Universal algebra is the study of classes of algebraic theories and their models of algebra in the category of sets. One of the aims of universal algebra is to extract the common elements of seemingly different types of algebraic structures such as groups, rings or lattices. Doing so one discovers general concepts, constructions, and results which unify and generalize the known special situations. The course will introduce the students to the basic concepts and theory of universal algebra. Applications of universal algebra can be found in logic through the interface of algebraic logic. Recent research in logic, recursive function theory, theory of automata and computer science has revealed that the preliminary notion of universal algebra could be fruitfully extended, for example to partial algebras and heterogeneous algebras.
Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. S. Burris and H.P. Sankappanavar. *A Course in Universal Algebra*. Springer New York, 1981
2. Ralph N. McKenzie, George F. McNulty, and Walter F. Taylor. *Algebras, Lattices, Varieties*. American Mathematical Society, 2017

ABM−756: Category Theory Category theory is the language of much of modern mathematics. It starts from the observation that the collection of all mathematical structures of a certain kind may itself be viewed as a mathematical object-a category. The main theme will be universal properties in their various manifestations, one of the most important uses of categories in mathematics. Category theory formalizes mathematical structure and its concepts in terms of a collection of objects and of arrows (also called morphisms). A category has two basic properties: the ability to compose the arrows associatively and the existence of an identity arrow for each object. This course is also essential for Computer science students because category theory is a mathematical approach to the study of algebraic structure that has become an important tool in theoretical computing science, particularly for semantics-based research. The aim of this course is to teach the basics of category theory, in a way that is accessible and relevant to computer scientists. The emphasis is on gaining a good understanding of the basic definitions, examples, and techniques, so that students are equipped for further study on their own of more advanced topics if required.
Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. Steve Awodey. *Category Theory*. 2nd edition. Oxford University Press, 2010

ABM−757: Inverse Semigroups: The Theory of Partial Symmetries Symmetry is one of the most important organizing principles in the natural sciences. The mathematical theory of symmetry has long been associated with group theory. But there are aspects of symmetry which are more faithfully represented by a generalization of groups called inverse semigroups. The theory of inverse semigroups is described from its origins in the foundations of differential geometry through to its most recent applications in combinatorial group theory, and the theory of tilings. The applications of inverse semigroups lie in algebraic geometry and quantum groups. This course will serve not only the students studying mathematics but is also useful for the students of symmetry in the physical sciences.
Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. Stephen Lipscomb. *Symmetric Inverse Semigroups*. American Mathematical Society, 1996
2. Mark Lawson. *Inverse semigroups: The theory of Partial Symmetries*. World Scientific, 1998

ABM−758: Advanced Topics in Group Theory This module provides an introduction to advanced group theory. The aim of this course is to study some advanced topics in group theory and applications thereof. Typically the lecturer will focus either on geometric and combinatorial group theory or on the theory and applications of linear representations of finite groups. In particular, the projective special linear groups over small fields provide a rich vein of interesting cases on which to hang the general theory.
Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. Derek J.S. Robinson. *A course in the theory of groups*. 2nd edition. Springer Science & Business Media, 1996

ABM−759: Representation Theory of Algebraic Groups Algebraic groups. Orthogonal and

symplectic groups; Multilinear algebra; symmetric and exterior powers; Algebraic homomorphisms; Basic notions from representation theory; Representations of tori; Irreducible and semisimple representations; Isotypic components; The Lie algebra of an algebraic group; Representations of SL2 and sl2; Solvable and nilpotent Lie algebras; Semisimple Lie algebras; Representations of sl3; The root system associated with a semisimple Lie algebra; Root systems and weights; Representations of semisimple groups; The Weyl Character Formula and Freudenthal's Multiplicity Formula.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. J. Humphreys. *Introduction to Lie algebras and Representation Theorys*. Springer Science & Business Media, 1972

ABM−760: Algebraic Geometry Review of commutative ring theory, Hilbert's basis theorem, Affine varieties, The Zariski topology, Irreducible varieties, Hilbert's Nullstellensatz, The Coordinate ring, The Spectrum of a ring, Projective spaces and projective varieties, Projective closure of an affine variety, Graded rings and homogeneous coordinate ring, Morphisms of affine and projective varieties, Veronese maps and varieties, Segre maps and product of varieties, Grassmannians, Dimension, Degree, The Hilbert function and Hilbert polynomial, Tangent space, Singular points, Intrinsic nature of Tangent space, Quasiprojective varieties, A basis for Zariski topology, Rings of regular functions, Rational maps, Dominant rational maps, Birational equivalence, Rational varieties, Resolution of singularities, Hypersurfaces.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. J. Harris. *Algebraic Geometry: A First Course, Graduate Texts in Mathematics*. Springer, 1992

ABM−761: Linear Algebraic Groups Definition of an algebraic group, connections with Lie groups, overview of the classification, connectedness, simple connectedness, relation with the Lie algebras, Irreducibility and connectedness, the identity component; subgroups and homomorphisms, Connectedness (and closedness) of a subgroup generated by a family of images of irreducible varieties. Quasi-projective varieties; algebraic actions: existence of a closed orbit, adjoint orbits of SL (2, C) and SL (2, R). Existence of a faithful linear representation of an algebraic group, The Jordan decomposition in algebraic groups, Unipotent groups, Lie's theorem, Commutative algebraic groups, Diagonalizable groups and tori, the tangent algebra, tangent space (using derivations), the differential of a morphism, the Lie algebra of an algebraic group. Semi-simple and reductive Lie algebras, the classification of semi-simple Lie algebras.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. T. A. Springer. *Linear algebraic groups*. Modern Bikhäuser Classics, Boston, MA: Birkhäuser Boston Inc., 2009

ABM−762: Applied Group Theory Groups and Symmetry, Symmetry in Quantum Mechanics, Introduction to representation theory, Survey of matrix groups: GL, SL, SO, SU, SP etc. The wonderful 2×2 matrix groups, Quaternions and octonions, Lie algebras from Lie groups, Harmonic oscillators: Symplectic and metaplectic groups, Conformal groups and conformal algebras, Clifford algebras and spinors, Superconformal and Superpoincare algebras, Structure of semisimple Lie algebras, Kac-Moody and affine Lie algebras, and beyond, Highest weight representations of semisimple Lie algebras, Induced representations, Unitary Representations of the Lorentz and Poincar´e groups, Representations of supersymmetry algebras, Nonlinear sigma models: Quantum field theories defined by group manifolds and

homogeneous spaces, Geometry and topology of Lie groups.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. Roger W. Carter et al. *Lectures on Lie Groups and Lie Algebras*. Volume 32. Cambridge University Press, 1995

ABM−763: Commutative Algebra Rings and Ring homomorphisms, Ideals and Quotients Rings, Zero Divisors, Nilpotent elements, Nilradical and Jacobson radical, Operations on Ideals, Modules and Module Homomorphisms, Direct sum and product of modules, Finitely generated modules, Exact Sequence, Tensor Product of Modules, Primary Decomposition, Noetherian Rings, Artinian Rings, Hilbert Functions, Dimension Theory of Noetherian Local Rings, Cohen-Macaulay Rings and Modules.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. M. F. Atiyah and I. G. Macdonald. *Introduction to Commutative*. Algebra-Wesley Publishing Company, 1969

ABM−764: Computational Algebra and Geometry Parametric equations, affine varieties, projection and elimination, Grobner basis, applications, ideals, primary decomposition, intersection of two ideals, solving systems of two polynomials, enumerative geometry, coordinate rings, projective varieties, graded algebras, Grobner-Shirshov basis, description of an algorithm, Lie algebras and their representations, affine Lie algebras and truncated bases, invariant rings, representations of the general linear group, the tropical semiring, polyhedral basics, tropical varieties.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. David A. Cox, John Little, and Donal O Shea. *Ideals, Varieties and Algorithms. An Introduction to Computational Algebraic Geometry and Commutative Algebra*. 3rd edition. Springer, 2007

ABM−765: Coxeter Groups Orthogonal transformations in a real Euclidean space, Groups generated by reflections, Coxeter groups, root systems, Crystallographic groups, Fundamental regions for Coxeter groups, Coxeter graphs, Classification of finite root systems, Classification of finite crystallographic Coxeter groups, Order and structure of irreducible Coxeter groups. Generators and relations of Coxeter groups, Affine Coxeter groups, Classification, Applications and connections with other fields.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. J. Humphreys. *Reflection Groups and Coxeter Groups*. 3rd edition. Cambridge University Press, 1990

ABM−766: Applied Algebra Properties of integers and modular arithmetic, Euclidean algorithm, factorization, the Chinese remainder theorem, Groups, subgroups and their properties, cyclic groups and public-key encryption schemes, Symmetry groups and their manifestation in nature, crystallographic groups, physical properties of molecules such as fullerene or other objects with symmetric structures, Isomophisms and automorphisms of groups, Cayley's theorem, Lagrange's theorem, Fermat's Little Theorem and its role in polynomial-time primality testing algorithms, Rings, ideals, fields and their properties, Polynomial rings, vector spaces, extension fields, Finite fields, Berlekamp's algorithm for factoring polynomials over finite fields, Hamming code, linear code and error-correcting codes such as Reed-Solomon codes.
Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. Roger W. Carter et al. *Lectures on Lie Groups and Lie Algebras*. Volume 32. Cambridge University Press, 1995

ABM−767: Representation Theory of Finite Groups Introduction to matrix representations, permutation representations and G-sets. Definition and basic properties of complex representations of a finite group. Maschke's Theorem, Characters and character tables. The special cases of: cyclic groups, abelian groups, Schur's Lemma, orthogonality of characters, the number of irreducible characters. Induced representations, Frobenius Reciprocity, double coset formula, methods of calculation. Transitive and 2-transitive permutation representations and their characters. Representations of symmetric Groups.
Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. B. Steinberg. *Representation Theory of Finite Groups*. Springer-Verlag, 2012

ABM−768: Lie Algebras Definitions and first examples. Classical Lie algebras. Ideals and homomorphisms. Nilpotent Lie algebras. Engel's theorem. Solvable Lie algebras. Lie's theorem. Jordan Chevalley Decomposition. Radical and semi simplicity. The Killing form and Cartan's criterion. The structure of semisimple Lie algebras. Complete reducibility and Weyl's theorem. Representation theory of the Lie algebra sl(2). Toral subalgebras and root systems. Integrality properties. Simple Lie algebras and irreducible root systems.
Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. J. Humphreys. *Introduction to Lie algebras and Representation Theorys*. Springer Science & Business Media, 1972
2. K. Erdmann and M.J. Wildon. *Introduction to Lie algebras*. Springer Science & Business Media, 2006
3. Brian C. Hall. *Lie Groups, Lie Algebras, and Representations. An Elementary Introduction*. Springer International Publishing, 2015

ABM−769: Formal Languages and Automata This course is an introduction to a school of mathematics that studies the interconnections between machines, languages and algebra. The course concerns the relations between very simple machines called automata, a class of languages called regular and algebraic objects called monoids.
Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. J. M. Howie. *Automata and languages*. 1st edition. Clarendon Press, Oxford, 1991
2. Mark Lawson. *Inverse semigroups: The theory of Partial Symmetries*. World Scientific, 1998

List of Elective Courses in Computational Mathematics

Computational Mathematics			
S. No.	**Course Code**	**Course Title**	**Credit Hours**
1.	COM−751	Advanced Numerical Analysis	03
2.	COM−752	Computational Methods in Fluid Dynamics	02+01
3.	COM−753	Numerical Methods for Partial Differential Equations	02+01
4.	COM−754	PDE−Constrained Optimization	03
5.	COM−755	Finite Element Methods for Turbulent Flows	02+01
6.	COM−756	Transport Modelling	02+01
7.	COM−757	Scientific Computing	02+01

Description of the Elective Courses in Computational Mathematics

COM−751: Advanced Numerical Analysis The course has the aim to discuss some aspects of eigenvalues computation, spectral Perturbation theory of matrices and set of matrices. In the first part of the course we shall discuss the eigenvalue problems, computation of singular values and $\mu-$values. In the second part of the course we shall consider some numerical methods to deal with the nonlinear equation of the form $f(x) = 0$. In the third part we shall discuss interpolation and approximation while numerical integration and differentiation is considered in the final part of this course.

Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. Walter Gautschi. *Numerical Analysis*. 2nd edition. Springer, 2012

COM−752: Computational Methods in Fluid Dynamics Computational Fluid Dynamics (CFD) is a valuable resource in engineering. Commercial CFD tools can be used to predict performance in a very short turn-around time. The course is an introduction to advanced CFD codes, covering aspects of geometry modeling, grid generation, solution strategy, and post-processing. Numerical methods for the incompressible Navier-Stokes equations are introduced with emphasis on accuracy evaluation and efficiency. Several turbulence closures are readily available in commercial CFD codes; a tutorial will be given on basic concepts in turbulence modeling and several examples will show the impact of the modeling assumptions.

Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. Joel H. Ferziger, Milovan Peric, and Robert L. Street. *Computational Methods for Fluid Dynamics*. 3rd edition. Springer, 2019

COM−753: Numerical Methods for Partial Differential Equations This course addresses graduate students of all fields who are interested in numerical methods for partial differential equations, with focus on a rigorous mathematical basis. Many modern and efficient approaches are presented, after fundamentals of numerical approximation are established. Of particular focus are a qualitative understanding of the considered partial differential equation, fundamentals of finite difference, finite volume, finite element, and spectral methods, and important concepts such as stability, convergence, and error analysis.

Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. Randall J. LeVeque. *Finite Difference Methods for Ordinary and Partial Differential Equations - Steady State and Time Dependent Problems*. SIAM, 2007

COM−754: PDE-Constrained Optimization Linear and non-linear partial differential equations (PDEs) constitute one of the most widely used mathematical framework for modelling various physical or technological processes, such as fluid flow, structural deformations, propagation of acoustic and electromagnetic waves among countless other examples. Improvement in such processes therefore requires modelling and solving optimization problems constrained with PDEs, and more generally convex and non-convex optimization problems in spaces of functions. This course is intended to concentrate on theory pertinent for analyzing optimization problems of this type and also fundamental numerical methods for solving these problems. We will mostly concentrate on the optimal control of processes governed with linear and semi-linear elliptic PDEs. The course is designed to study following issues: Analyze control-to-state operators for model control problems, Derive necessary and sufficient optimality conditions for optimal control problems with or without state constraints, Assess existence of solutions to model optimal control problems, Implement optimization algorithms on a computer, Apply optimization algorithms to model problems, Explain the basic properties of the relevant functional spaces, in particular Sobelov spaces.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. M. Hinze et al. *Optimization with PDE COnstraints, Mathematical Modelling: Theory and Applications*. Springer, 2009

COM−755: Finite Element Methods for Turbulent Flows This course is intended to cover advanced topics in numerical simulations such as Phenomenological and theoretical introduction to turbulence, microscales and integral scales. Reynolds' decomposition and Reynolds' averaging. Homogeneous turbulence. Turbulent shear flows: boundary layers, pipe flows and jets. Engineering calculation approaches: integral methods, mean field methods, and large-eddy simulations. Turbulent modelling: algebraic and transport models. Examples of numerical solutions of internal and external flows. Following topics are included in this course: Basic laws in fluid mechanics. Cartesian tensor notation, Dimensionless equations. Reynolds number, scales for length and time, Vorticity and elements of vortex dynamics, Characteristics of turbulence, Reynolds-averaged Navier-Stokes equation (RANS), Balance equations for the mean flow and the turbulent kinetic energy, Turbulence near solid surfaces. The mean velocity profile. Boundary layer equations, Free turbulent shear flows. Simplified momentum equations. Self-preservation, Turbulence modelling. Eddy viscosity, mixing-length, Reynolds stress models, Direct numerical simulation (DNS) and large-eddy simulation (LES).
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. Roland Schiestel. *Modelling and Simulations of turbulent flows*. John Wiley & Sons, 2008

COM−756: Transport Modelling The goal of this course is to gain experience in environmental modeling with or without uncertain parameters, as the main field for illustration and application, transport modeling has been chosen. Introductions to transport models: Examples of real-life applications, Mathematical Modeling of the Transport of Pollution, diffusion and dispersion, An Alternative Approach: Lagrangian Tracer Technique, Monte-Carlo method, Lagrangian versus Eulerian: pros and cons, Lagrangian simulation of advection and turbulent mixing, forward and reverse time diffusion approaches.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. C. Zheng and G.D. Bennett. *Applied Contaminant Transport Modelling*. 2nd edition.

Wiley, 2002
2. Z. Zlatev and I. Dimov. *Computational and Numerical Challenges in Environmental Modelling*. 2nd edition. Elsevier, 2006

COM−757: Scientific Computing This course provides an extensive hands-on experience of computer programming and the theory of Mathematical techniques useful for mathematicians. The course exposes students to the theoretical as well as the practical aspects of scientific and high performance computing. The widely used Python/Julia programming Language will be used for implementing various related applications within the domain of Mathematical sciences.
Pre-requisite: Scientific Computing is the pre-requisites for this course.
Recommended Books
1. John V. Guttag. *Introduction to Computation and Programming Using Python*. MIT Press, Cambridge, 2013

List of Elective Courses in Applied Mathematics

Applied Mathematics			
S. No.	**Course Code**	**Course Title**	**Credit Hours**
1.	APM−751	Fluid Dynamics	03
2.	APM−752	Viscous Fluids	03
3.	APM−753	Biofluid Mechanics	03
4.	APM−754	Non−Newtonian Fluid Mechanics	03
5.	APM−755	Theory of Fluids	03
6.	APM−756	Methods of Applied Mathematics	03
7.	APM−757	Perturbation Methods	03
8.	APM−758	Geophysical Fluid Dynamics	03
9.	APM−759	Theory of Relativity	03
10.	APM−760	Theory of Quantum Physics	03
11.	APM−761	Advanced Computational Physics	03
12.	APM−762	Mathematical Economics−I	03
13.	APM−763	Mathematical Economics−II	03
14.	APM−764	Econometric Analysis	03
15.	APM−765	Image Processing	03
16.	APM−766	Matrix Computation−I	03
17.	APM−767	Matrix Computation−II	03
18.	APM−768	Function of Matrices−I	03
19.	APM−769	Function of Matrices−II	03
20.	APM−770	Advanced Analytical Dynamics	03
21.	APM−771	Theory of Bifurcation and Chaos	03
22.	APM−772	Nonlinear Control Systems	03
23.	APM−773	Optimal Control	03
24.	APM−774	Nonlinear Dynamical Systems	03
25.	APM−775	Robot Manipulator Control	03
26.	APM−776	Numerical Solution of Stochastic Differential Equations	02+01
27.	APM−777	Computational Astrophysics	03
28.	APM−778	Theory of Differential Equations	03
29.	APM−779	Uncertainty Theory	03

Description of the Elective Courses in Applied Mathematics

APM−751: Fluid Dynamics The course will increase the basic level of the students in the field of fluid mechanics and will grow their knowledge from both mathematical point of view and also from application point of view. Also this course will help those students who wish to undertake advanced studies in fluid mechanics. The contents include: Introduction and Fundamental Concepts. Basic Definitions, Fluid as a Continuum, Velocity Field, Timelines, Pathlines, Streaklines, Streamlines, Stress Field, Viscosity, Newtonian and Non-Newtonian Fluids, Surface Tension, Description and Classification of Fluid Motion, Fluid Statics: The Basic equation of Fluid Statics, The Standard Atmosphere, Pressure Variation in a Static Fluid, Buoyancy and Stability, Introduction to Differential Analysis of Fluid Motion Conservation of Mass, Stream Function for Two-Dimensional Incompressible Flow, Motion of a Fluid Particle, Momentum Equation Incompressible inviscid Flow: Euler's and Bernouli's Equations, Irrotational Flow Dimensional Analysis and Similitude Nondimensionalzing the

Basic Differential Equations.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. Alexander J. Smits. *A Physical Introduction to Fluid Mechanics*. 2nd edition. Wiley, 2000
2. Fox and McDonald. *Introduction to Fluid Mechanics*. 8th edition. John Wiley & Sons, Inc., 2011

APM−752: Viscous Fluids Some examples of viscous flow phenomena; properties of fluids; boundary conditions. Equation of continuity; the Navier stokes equations; the energy equation; boundary conditions; orthogonal coordinate systems; dimensionless parameters; velocity considerations; two dimensional considerations, and the stream functions. Coutte flows; Poiseuille flow; unsteady duct flows; similarity solutions; some exact analytic solution from the papers. Introduction; laminar boundary layers equations; similarity solutions; two dimensional solutions; thermal boundary layer. Some exposure will also be given from the recent literature appearing in the journals. The concept of small disturbance stability; linearized stability; parametric effects in the linear stability theory; transition to turbulences. Boundary layer equation in plane flow; general solution and exact solutions of the boundary layer equations.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. Tasos C. Papanastasiou, Georgios C. Georgiou, and Andreas N. Alexandrou. *Viscous Fluid Flow*. CRC Press, 2000
2. Ascher H. Shapiro and Ain A. Sonin. *Advanced Fluid Mechanics Problems*. Lecture Note MIT

APM−753: Biofluid Mechanics This course elaborates on the application of fluid mechanics principles to major human organ systems. The course is an introduction to physiologically relevant fluid flow phenomena, underlying physical mechanisms from an engineering perspective. The focus of the course is on the integration of various fluid mechanics concepts to address relevant problems of the human body's systems.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. C. Ross Ethier and Craigg A. Simmons. *Introductory Biomechanics*. Cambridge University Press, 2007
2. C. Kleinstreuer. *Biofluid Dynamics: Principles and Applications*. CRC Press, Taylor & Francis Group, 2006

APM−754: Non-Newtonian Fluid Mechanics Newtonian versus non-Newtonian behavior. Review of Newtonian fluid dynamics. Elementary constitutive equations and their use in solving fluid dynamics problems. Nonlinear viscoelastic constitutive equations and their use in solving fluid dynamics. Or Classification of Non-Newtonian Fluids, Rheological formulae (Time-independent fluids, Thixotropic fluids and viscoelastic fluids), Variable viscosity fluids, Cross viscosity fluids, The deformation rate, Viscoelastic equation, Materials with short memories, Time dependent viscosity. The Rivlin-Ericksen fluid, Basic equations of motion in rheological models. The linear viscoelastic liquid, Couette flow, Poiseuille flows. The current semi-infinite field, Axial oscillatory tube flow, Angular oscillatory motion, Periodic transients, Basic equations in boundary layer theory, Orders of magnitude, Truncated solutions for viscoelastic flow, Similarity solutions, Turbulent boundary layers, Stability analysis.
Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. John Harris. *Theology and Non-Newtonian Flow*. Longman Inc, New York, 1997

APM−755: Theory of Fluids Euler's equation of motion, Viscosity, Navier-Stoke's equation and exact solutions, Dynamical similarity and Reynold's number, Turbulent flow, Boundary layer concept and governing equations, Reynold's equations of turbulent motion. Magnetohydrodynamics, MHD equations, Fluid Drifts, Stability and equilibrium problems.
Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. G. K. Batchelor. *Fluid Dynamics*. Cambridge University Press, 1967
2. John Harris. *Theology and Non-Newtonian Flow*. Longman Inc, New York, 1997

APM−756: Methods of Applied Mathematics This is a first-year course for all incoming PhD and Master Students interested in pursuing research in applied mathematics. It provides a concise and self-contained introduction to advanced mathematical methods, especially in the asymptotic analysis of differential equations. Topics include scaling, perturbation methods, multi-scale asymptotics, transform methods.
Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. G. I. Barenblatt. *Cambridge Texts in Applied Mathematics. Scaling, Self-similarity, and Intermediate Asymptotics: Dimensional Analysis and Intermediate Asymptotics*. Cambridge University Press, 1996

APM−757: Perturbation Methods Exact or numerical techniques are not the only way to solve problems or understand their solutions. This course describes the machinery of asymptotic analysis which can be applied to the solution of physical problems. The syllabus includes: I. Asymptotic series II. Solution of algebraic systems III. Integrals IV. Differential equations V. Matched asymptotic VI. Multiple scales VII. Improvement of series Special emphasis will be given to applying the techniques to problems of physical relevance (e.g. analysis of wave dispersion relations, amplitude expansions for forming patterns, dynamics of nonlinear oscillators, fluid boundary layers). Grading: Assessment will involve coursework (homework problems) and examination.
Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. Ali H. Nayfeh. *Perturbation Methods*. John Wiley & Sons, 2004
2. E. J. Hinch. *Perturbation Methods*. Cambridge University Press, 2000

APM−758: Geophysical Fluid Dynamics The recent rapid development of Geophysical Fluid Dynamics includes the following very important, challenging and multidisciplinary set of problems: Earth system modelling, Predictive understanding of climate variability (emerging new science!), Forecast of various natural phenomena (e.g., weather), Natural hazards, environmental protection, natural resources, etc. The idea behind offering this course is to provide a strong foundation to the students who are willing to conduct their research in the topics related to the field of geosciences.
Pre-requisite: There is no pre-requisites for this course.

Recommended Books

1. James C. McWilliams. *Fundamentals of geophysical fluid dynamics*. Cambridge University Press, 2011
2. Benoit Cushman-Roisin and Jean-Marie Beckers. *Introduction to geophysical fluid dynamics*. Academic Press, 2011

APM−759: Theory of Relativity This course covers the basic principles of Einstein's general

theory of relativity intuitively and rigorously. The course contents include: Introduction to Newtonian gravity, introduction of spacetime structure for Special Relativity, differential geometry concepts to study the curved spacetime of General Relativity, Einstein's field equations, Gravitational waves solution, Schwarzschild solution, cosmological models and experimental tests of general relativity.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books
1. Bernard Schutz. *A First Course in General Relativity*. Cambridge University Press, 2009
2. James B. Hartle. *Pearson, Gravity: An Introduction to Einstein's General Relativity*. Cambridge University Press, 2003

APB−760: Theory of Quantum Physics This course covers the basics of python programming, using computers to solve complex physics problems. We will study topics including dangers in computer arithmetic, Algorithms, functions as arguments, Maxima/minima, Numerical Integration, Numerical differentiation, Numerical solution of Differential equations, Solution of system of linear equations and eigenvalue problems. All of the topics will have associated physics problems to solve using python available tools.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books
1. Mark Newman. *Computational Physics*. Create Space Independent Publishing Platform, 2012
2. Rubin H. Landau et al. *Computational Physics: Problem Solving with Computers*. 2nd edition. Wiley, 1997

APM−761: Advanced Computational Physics The Python programming language is an excellent choice for learning, teaching, or doing computational physics. It is a well-designed, modern programming language that is simultaneously easy to learn and very powerful. It includes a range of features tailored for scientific computing, including features for handling vectors, inverting and diagonalizing matrices, performing Fourier transforms, making graphs, and creating 3D graphics.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books
1. Rubin H. Landau, Manuel J Paez, and Cristian C. Bordeianu. *Computational Physics: Problem Solving with Python*. 3rd edition. John Wiley & Sons, 2015
2. Rubin H. Landau et al. *Computational Physics: Problem Solving with Computers*. 2nd edition. Wiley, 1997

APM−762: Mathematical Economics−I Mathematical tools are an important part of theoretical economic analysis. This course gives students a working knowledge of static and comparative Static and optimization techniques applied in economics. Topics include Static Analysis, comparative Static Analysis, Optimization without constraints, optimization with equality constraints, optimization with nonlinear constraints, maximum-value functions and envelop theorem, duality and envelop theorem. The knowledge of matrix algebra, calculus and differential equations will equip students to solve complex economic models. All mathematical techniques are illustrated with mainstream theoretical applications such as consumer theory and the neoclassical theory of optimal growth.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books
1. Alpha C. Chiang and Kevin Wainwright. *Fundamental Methods of Mathematical Economics*. 4th edition. McGraw Hill Education (India) Private Limited, 2013

2. Carl P. Simon and Lawrence Blume. *Mathematics for Economists*. Norton, 1994

APM−763: Mathematical Economics−II Mathematical tools are an important part of theoretical economic analysis. This course gives students a working knowledge of dynamic optimization techniques applied in economics. Topics include, Dynamics and Integral Calculus,, Continuous Time: First order differential equation, higher order differential equations, Discrete Time: First order difference equation, higher order difference equations, Simultaneous differential and difference equations. Knowing these tools will equip students to solve complex economic models such as Domar Growth model, Solow growth model, Neoclassical optimal growth model, dynamics of Market price, dynamic stability of equilibrium, the interaction of inflation and unemployment and optimal control theory.
Pre-requisite: Mathematical Economics−I is the pre-requisites for this course.
Recommended Books

1. Carl P. Simon and Lawrence Blume. *Mathematics for Economists*. Norton, 1994
2. Alpha C. Chiang and Kevin Wainwright. *Fundamental Methods of Mathematical Economics*. 4th edition. McGraw Hill Education (India) Private Limited, 2013

APM−764: Econometric Analysis This course introduces the use of econometrics to estimate, inference, and identify economic relationships using linear regression models. The course focus is paid to the econometric theory and its applications to real world problems, and the interpretation of the estimation results. The course includes, a review on statistics and matrix algebra, methods of estimations of simple and multiple linear regression, tests of stability, violation of the assumptions of basic regression model and introduction to time series analysis. The course will include the use of EViews or R, a standard software for econometric and statistical analysis.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. G. S. Maddala and Kajal Lahiri. *Introduction to Econometrics*. 4th edition. Wiley India, 2009

APM−765: Image Processing This course provides an introduction to image processing concepts, algorithms and techniques. This course contents can be divided into three phases. Initially, Fundamental concepts of image representation, digitizing existing images and camera models are introduced which serve as baseline for upcoming high level image processing algorithms. In the second phase, high-level concepts such as linear and non-linear filtering, image denoising and convolution based spatial filters are introduced. In the final phase, algorithms related to digital image processing are introduced and applications of image processing including morphological operations, feature estimation (such as lines, edges and corners) and segmentation of image are introduced. Few lab sessions will be conducted as well to provide students the confidence to implement any image processing algorithms using any language of their choice.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. K. R. Castleman. *Digital Image Processing*. Pearson Education India., 1996

APM−766: Matrix Computation−I The course an in-depth study of numerical linear algebra and the matrix computations that arise in solving linear systems, least squares problems, and eigenvalue problems for dense and sparse matrices. It will cover many of the fundamental concepts related to matrix multiplication, matrix analysis, general linear systems, special linear systems and orthogonalization and least squares. The course is designed for those who wish to use matrix computations in their own research.
Pre-requisite: Linear Algebra−I & II are the pre-requisites for this course.

Recommended Books

1. Golub and Van Loan. *Matrix Computations*. 4th edition. JHU Press, 2013

APM−767: Matrix Computation−II This course is designed for 2nd and above semester master students and for PhD students. This course is an in-depth study of numerical linear algebra and the matrix computations that arise in solving linear systems, least squares problems, and eigenvalue problems for dense and sparse matrices. It will cover advanced topics related to modified least square problems and methods, asymmetric eigenvalue problems, symmetric eigenvalue problems, large sparse eigenvalue problems and large sparse linear system problems. The course is designed for those who wish to use matrix computations in their own research.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. Golub and Van Loan. *Matrix Computations*. 4th edition. JHU Press, 2013

APM−768: Function of Matrices−I This course is designed for senior undergraduate and first semester master students. It will cover many of the fundamental concepts related to theory of matrix functions, Applications, sensitivity analysis, conditioning techniques for general functions and matrix sign Function. The course is designed for those who wish to use matrix computations in their own research.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. Nicholas J. Higham. *Function of Matrices, theory and computation*. SIAM, 2008

APM−769: Function of Matrices−II This course is designed for 2nd semester master students and for PhD students. It will cover topics like matrix square root, matrix pth root, polar decomposition, Schur-Parlett algorithm, matrix exponential, matrix logarithm, matrix cosine and sine, function of matrix times vectors. The course is designed for those who wish to use matrix computations in their own research.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. Nicholas J. Higham. *Function of Matrices, theory and computation*. SIAM, 2008

APM−770: Advanced Analytical Dynamics The aim of this course is to advance the student's knowledge of classical mechanics. The mathematical structure of mechanics is treated. This course reviews momentum and energy principles, and then covers the following topics: Hamilton's principle and Lagrange's equations; three-dimensional kinematics and dynamics of rigid bodies; steady motions and small deviations therefrom; gyroscopic effects; and causes of instability; free and forced vibrations of lumped-parameter and continuous systems; nonlinear oscillations and the phase plane; nonholonomic systems; and an introduction to wave propagation in continuous systems.
Pre-requisite: Classical Mechanics is the pre-requisites for this course.
Recommended Books

1. Haim Baruh. *Analytical Dynamics*. CRC Press, 2014

APM−771: Theory of Bifurcation and Chaos Modelling with nonlinear systems of ODE's. Stability and bifurcation theory including the Hopf bifurcation and limit cycles. Homoclinic & heteroclinic orbits and Mel'nikov theory. Stability, bifurcation theory and chaos in I-dimensional Maps. Period doubling. Feigenbaum's approach to chaos. Properties of chaos. The Lorenz Equation.
Pre-requisite: Classical Mechanics is the pre-requisites for this course.
Recommended Books

1. Sauer Alligood and Yorke. *Chaos: An Introduction to Dynamical Systems*. Springer, 1997
2. Ya. B. Pesin, Yakov B. Pesin, and Vaughn Climenhaga. *Lectures on Fractal Geometry and Dynamical Systems*. American Mathematical Society, 2009

APM−772: Nonlinear Control Systems Methods for analysis and design of nonlinear systems, with an emphasis on nonlinear control systems. The course includes: Mathematical models of nonlinear systems, and fundamental differences between the behavior of linear and nonlinear systems. Equilibrium points, limit cycles and general invariant sets. Phase plane analysis, Lyapunov stability, Input-to-state stability, Input-Output Stability, and Passivity analysis. Nonlinear control design, including Lyapunov-based control, Energy-based Control, Cascaded control, Passivity-based control, Input-Output linearization, and Backstepping.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. Alberto Isidori. *Nonlinear Control Systems*. Springer, 1995
2. S. Engelberg. *A Mathematical Introduction to Control Theory*. 2nd edition. Imperial College Press, 2015

APM−773: Optimal Control The theory of optimal control is to evaluate the control signals that satisfy some physical constraints and minimize or maximize some performance measure. Calculus of variation as well as State Variable methods are discussed and reviewed. Pontryagin's maximum principle and dynamic programming to problems of optimal control theory. Interactive numerical techniques for finding optimal trajectories.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. E. K. Donald. *Optimal control theory - an introduction*. Prentice Hall, 1970
2. Lawrence C. Evans. *An Introduction to Mathematical Optimal Control Theory. Lecture Notes*. University of California, Berkeley

APM−774: Nonlinear Dynamical Systems Nonlinear dynamics with applications. Intuitive approach with emphasis on geometric thinking, computational and analytical methods. Extensive use of demonstration software. Topics include: Bifurcations, Phase plane, Nonlinear coupled oscillators in biology and physics, Perturbation, averaging theory, Parametric resonances, Floquet theory, Relaxation oscillations, Hysterises, Phase locking, Chaos, Lorenz model, iterated mappings, period doubling, renormalization, Hamiltonian systems, area preserving maps and KAM theory.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. Steven H. Strogatz. *Nonlinear Dynamics and Chaos*. 2nd edition. Avalon Publishing, 2014
2. P. G. Drazin. *Nonlinear Systems*. Cambridge University Press, 1992
3. H-O. Peitgen, H. Jurgens, and D. Saupe. *Chaos and Fractals*. Springer, 2004

APM−775: Robot Manipulator Control This course focuses on the application of control theory in robotics. Topics to be covered include: review of classical and modern control design methods such as PID control, State feedback, optimal control, adaptive control, and hybrid system control; control of mobile Robots; control of robot manipulators.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. M. Mataric. *The Robotics Primer*. MIT Press, 2007
2. K. J. Astrom and R. M. Murray. *Feedback Systems: An Introduction for Scientists and Engineers*. Princeton University Press, 2010

3. M. Neil and L. Frank. *Robot Manipulator Control Theory and Practice*. 2nd edition. Marcel Dekker Inc., 2004

APM−776: Numerical Solution of Stochastic Differential Equations Ito calculus for stochastic integrals, Stratonovich calculus, stochastic differential equations, Fokker-Planck equations, Stochastic Taylor expansion: Euler scheme, Milstein scheme and higher order schemes, strong order of convergence, weak order of convergence, Ito-backward methods, Kernel estimates, Variance reduction and applications in financial mathematics (option pricing) and environmental modelling (pollution transport).
Pre-requisite: There is no pre-requisites for this course.
Recommended Books
1. Peter E. Kloeden and Eckhard Platen. *Numerical Solution of Stochastic Differential Equations*. Springer, 1992
2. C.W. Gardiner. *Handbook of Stochastic Methods: for Physics, Chemistry and the Natural Sciences*. Springer, 2004

APM−777: Computational Astrophysics Computational astrophysics opens new windows in the way we perceive and study the heavens. This rapidly growing new discipline in astronomy combines modern computational methods, advanced algorithms for both simulations and data analysis, original software implementations to discover new phenomena, and to make predictions in astronomy and cosmology. We will cover both theoretical and computational aspects of astrophysics in this course using and extending our knowledge in astronomy, physics, mathematics and computations.
Pre-requisite: Calculus−I, II & III, Linear Algebra *I* & II, Differential Equation and Python are pre-requisites for this course.
Recommended Books
1. Wolfram Schmidt and Marcel Völschow. *Numerical Python in Astronomy and Astrophysics: A Practical Guide to Astrophysical Problem Solving (Undergraduate Lecture Notes in Physics)*. Springer, 2021
2. Simon Portegies Zwart and Steve McMillan. *Astrophysical Recipes: The Art of AMUSE*. 1. IOP Publishing, 2018
3. Brian Koberlein and David Meisel. *Astrophysics Through Computation: With Mathematica Support*. 1st edition. Cambridge University Press, 2013

APM−778: Theory of Differential Equations Most "real life" systems that are described mathematically, be they physical, biological, financial or economic, are described by means of differential equations. Our ability to predict the way in which these systems evolve or behave is determined by our ability to model these systems and find solutions of the equations explicitly or approximately. Every application and differential equation presents its own challenges, but there are various classes of differential equations, and for some of these there are established approaches and methods for solving them. Topics covered are: first order ordinary differential equations (ODEs), higher order ODEs, systems of ODEs, Power series solutions of ODEs, interpretation of solutions, Fourier analysis and solution of linear partial differential equations using the method of separation of variables, wellposed.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books
1. Dennis G. Zill and Michael R. Cullen. *Differential Equations and Boundary value problems*. 7th edition. Cengage Learning, 2008
2. Walter A. Strauss. *Partial Differential Equations. An Introduction*. 2nd edition. Wiley, 2007

APM−779: Uncertainty Theory Uncertainty theory is a branch of mathematics based on

normality, monotonicity, self-duality, countable subadditivity, and product measure axioms. Mathematical measures of the likelihood of an event being true include probability theory, capacity, fuzzy logic, possibility, and credibility, as well as uncertainty. In this course, we aim to cover uncertain measures, uncertain random variables, uncertain Calculus, and differential equations, with applications to physics and finance.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. Lu Boding. *Uncertainty Theory. A Branch of Mathematics for Modeling Human Uncertainty*. Springer, 2020
2. Kai Yao. *Uncertain Differential Equations*. Springer, 2016

List of Elective Courses in Analysis

Analysis			
S. No.	**Course Code**	**Course Title**	**Credit Hours**
1.	ANA−751	Function Spaces	03
2.	ANA−752	Advanced Functional Analysis	03
3.	ANA−753	Evolution Equations	03
4.	ANA−754	Lebesgue Integration	03
5.	ANA−755	Distribution Theory and Sobelov Spaces	03
6.	ANA−756	Analysis of Partial Differential Equations	03
7.	ANA−757	Theory of Interpolation Spaces	03
8.	ANA−758	Banach Space Theory	03

Description of the Elective Courses in Analysis

ANA−751: Function Spaces The course aims to provide a deeper understanding of specialized functions spaces, which central to several applications. Lebesgue sequence spaces and related theorems. Hardy and Hilbert inequalities. Essentially bounded functions, Lebesgue spaces $(p > 1)$, Embedding's, approximation in Lebesgue spaces, Duality, Reflexivity, weak convergence and continuity of translation operator, weighted Lebesgue spaces, Isometries, Lebesgue spaces $(0 < p < 1)$; Weak Lebesgue spaces, Convergence in Measure, Interpolation and Normability; Lorentz space and Normability, Completeness, Separability, Duality, Lorentz sequence spaces; Variable Lebesgue space Definition and their properties. Some differences between constant exponent and variable exponent spaces. Grand Lebesgue space, Amalgam and Herz type spaces with variable exponents

Pre-requisite: Functional Analysis is the pre-requisites for this course.

Recommended Books

1. David R. Adams and Lars I. Hedberg. *Function Spaces and Potential Theory*. Springer, 1996
2. Hans Triebel. *Theory of Function Spaces*. Springer, 2000

ANA−752: Advanced Functional Analysis The aim of the course is to analysis background Completeness, separability, compactness, and duality; Lebesgue spaces: completeness, dense sets, linear functionals and weak convergence; Distributions and distributional derivatives; Sobolev spaces: mollifications and weak derivatives, completeness, Friedrichs inequality, star-shaped domains and dense sets, extension of functions with weak derivatives; Embedding of Sobolev spaces into Lebesgue spaces: Poincare inequality, Reillich-Kondrachov-Sobolev theorems on compactness; Traces of functions with weak derivatives; Dirichlet boundary value problems for elliptic PDE's, Fredholm Alternative (uniqueness implies existence); Smoothness of weak solutions: embedding from Sobolev spaces into spaces of Hölder continuous functions, interior regularity of distributional solutions to elliptic equations with continuous coefficients

Pre-requisite: Functional Analysis is the pre-requisites for this course.

Recommended Books

1. David R. Adams and Lars I. Hedberg. *Functional Analysis in Applied Mathematics and Engineering*. CRC Press, 2000
2. Kosaku Yosida. *Functional Analysis*. Springer, 1980

ANA−753: Evolution Equations The course mainly focuses on the functional analytic method for solving the time dependent ordinary and partial differential equations with values in

Banach spaces. The key contents include: Banach space valued functions of one real variable. Linear problems with bounded operators. Spectrum, resolvent, and spectral properties of linear operators in Banach spaces. Strongly continuous semigroups and their infinitesimal generators. The Hille-Yosida Theorem. Non homogeneous Cauchy problems. Applications to Cauchy problems for linear evolutionary PDEs. Sectorial operators and analytic semigroups. Asymptotic behavior in homogeneous and in nonhomogeneous problems. Applications to PDEs of parabolic type.
Pre-requisite: Functional Analysis is the pre-requisites for this course.
Recommended Books
1. Songmu Zheng. *Nonlinear Evolution Equations*. CRC Press, 2004

ANA−754: Lebesgue Integration This is an introduction to measure and integration theory. It is particularly suitable for further studies in analysis, probability and statistics. The key contents includes: Lebesgue measure on the line: outer measure, measurable sets, non-measurable sets, measurable functions; Lebesgue integration on the line; Monotone convergence theorem, Fatou's Lemma, dominate convergence theorem; Almost everywhere convergence, convergence in measure, Egoroff's theorem; Differentiation, absolute continuity, derivatives of integrals; General measure and integration theory; Signed measures, Hahn decomposition theorem, Jordan decomposition; Radon-Nikodym theorem, Lebesgue decomposition. Outer measure, extension of measures, Lebesgue-Stieltjes measures; Product measures, Fubini and Tonelli theorems; L-p-spaces.
Pre-requisite: Real Analysis is the pre-requisites for this course.
Recommended Books
1. H. S. Bear. *A Primer of Lebesgue Integration*. Academic Press, 1995
2. J. Franks. *A (Terse) Introduction to Lebesgue Integration*. American Mathematical Society, 2009

ANA−755: Distribution Theory and Sobelov Spaces The course is aimed to introduce the Distribution Theory and Sobelov spaces in such a way that they serve as foundations of the theory of Partial differential equations. The key contents include: The spaces D and D', Regular distributions, Derivatives and multiplications with smooth functions, Localizations, the spaces ε', Schwartz space and Fourier Transform, The spaces $W^{k,p}(R^n)$ and $H^s(R^n)$, Embeddings, Extensions, Traces, Spectral theory in Hilbert spaces and Banach Spaces, Applications of Distribution and Sobelov spaces.
Pre-requisite: Functional Analysis is the pre-requisites for this course.
Recommended Books
1. Leoni Giovanni. *A First Course in Sobolev Spaces*. American Mathematical Society, 2009

ANA−756: Analysis of Partial Differential Equations A rigorous introduction to the theoretical underpinnings of the basic methods and techniques in the modern theory of PDEs. It is aimed at math majors, but will also be useful to some students in the sciences, engineering and economics who feel the need for a deeper understanding of the theory of PDEs. The emphasis is on the exposure to a number of different methods of solution of PDEs and their connection to physical phenomena modeled by the equations. The goals include both learning to solve some basic types of PDEs as well as to understand the motivation behind and inner workings of the techniques involved.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books
1. Lawrence C. Evans. *Partial differential equations*. 2nd edition. American Mathematical Society, 2010

ANA−757: Theory of Interpolation Spaces This course gives an introduction to the theory of interpolation spaces. The topics include general interpolation methods, real and complex interpolation methods, equivalence theorems, reiteration theorems, Wolff's theorem, duality theorems, compactness theorems, connections between real and complex methods, interpolation of Lebesgue spaces, interpolation of Lorentz spaces, interpolation of semi-groups of operators, interpolation of non-linear operators, interpolation of subspaces and factor spaces, Besov and Sobolev spaces and their interpolation properties.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. Jöran Bergh and Jörgen Löfström. *Interpolation Spaces: An Introduction*. Springer Science & Business Media, 2012

ANA−758: Banach Space Theory This course gives an introduction to classical Banach space theory. The topics include Schauder bases, equivalence of bases and basic sequences, construction of basic sequences, isomorphic structure of and, complemented subspaces of and , unconditional bases, bases and duality, nonreflexive spaces with unconditional bases, characterization of real spaces, weakly compact subsets of, Dunford-Pettis property, weakly compact operators on, Haar basis in , subspaces of, Maurey-Nikishin factorization theorems, factoring through Hilbert spaces, absolutely summing operators, Grothendieck's and Pietsch's theorems, approximation numbers of operators between Banach spaces, Weyl's inequality, Schatten-von Neumann classes and eigenvalues.
Pre-requisite: There is no pre-requisites for this course.
Recommended Books

1. F. Albiac and N. J. Kalton. *Topics in Banach Space Theory*. 2nd edition. Springer-Verlag, 2016
2. R. E. Megginson. *An Introduction to Banach Space Theory*. Springer, 1998